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# Multi-angle effects in collective supernova neutrino oscillations

**A Esteban-Pretel[1], S Pastor[1], R Tomàs[1], G G Raffelt[2], G Sigl[3]**

[1] AHEP Group, Institut de Física Corpuscular, CSIC/Universitat de València, Edifici Instituts d'Investigació, Apt. 22085, 46071 València, Spain

[2] Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), Föhringer Ring 6, 80805 München, Germany

[3] II. Institut für Theoretische Physik, Universität Hamburg, Luruper Chaussee 149, 22761 Hamburg, Germany

E-mail: andreu.esteban@ific.uv.es, sergio.pastor@ific.uv.es, ricard@mppmu.mpg.de, raffelt@mppmu.mpg.de, sigl@mail.desy.de

**Abstract.** We study two-flavor collective neutrino oscillations in the dense-neutrino region above the neutrino sphere in a supernova (SN). The angular dependence of the neutrino-neutrino interaction potential causes "multi-angle" effects that can lead either to complete kinematical decoherence in flavor space or only to small differences between different trajectories. This nonlinear system switches abruptly between "self-maintained coherence" and "self-induced decoherence" among the angular modes, depending on the strength of the deleptonization flux. For a realistic SN the quasi single-angle behavior is probably typical, simplifying the numerical treatment and probably allowing for the survival of observational features of flavor oscillations.

The neutrinos streaming off a collapsed supernova (SN) core are so dense near the neutrino sphere that they produce a significant refractive effect for each other, leading to collective oscillation effects. The practical importance of these nonlinear phenomena was only recently recognized and studied in a series of papers [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. An introduction was given at this conference by Eligio Lisi [16].

The SN neutrino fluxes are thought to obey the hierarchy $F_{\nu_e} > F_{\bar\nu_e} > F_{\nu_x}$ where $\nu_x$ stands for any of $\nu_\mu$, $\nu_\tau$, $\bar\nu_\mu$, and $\bar\nu_\tau$. In other words, there is an excess of $\nu_e\bar\nu_e$ pairs relative to $\nu_x\bar\nu_x$. In an inverted hierarchy situation, this pair excess converts collectively into $\nu_x\bar\nu_x$ pairs, a process that does not violate flavor-lepton number and thus does not require mixing: it could also proceed as an ordinary pair annihilation process. Neutrino refraction causes this pair process to proceed collectively and very fast, almost independently of the mixing angle, i.e., we have to do with a collective "speed-up effect" [1, 2]. The unpaired $\nu_e$ excess flux from deleptonization is conserved [5]. In the adiabatic limit, it is the low-energy part of the $\nu_e$ spectrum that survives, leading to a step-like feature in the $\nu_e$ spectrum (a "spectral split" [9, 10] caused by a "step-wise spectral swapping" [4, 12]). All of these effects happen in the dense-neutrino region that typically extends from the neutrino sphere out to a few hundred kilometers. If the ordinary MSW resonances occur at larger radii, the collective phenomena and subsequent MSW transformations are independent, the former producing the initial condition for the latter. For an inverted hierarchy the only effect of matter in the collective-transformation region is a decrease of the effective mixing angle [5, 8]. Conversely, if the matter density profile is very shallow, the

ordinary MSW effect can prepare the initial condition for the collective effects [14].

Mixed neutrinos are described by matrices of density $\rho_{\mathbf{p}}$ and $\bar{\rho}_{\mathbf{p}}$ for each (anti)neutrino mode. The diagonal entries are the usual occupation numbers whereas the off-diagonal terms encode phase information. The equations of motion are $\mathrm{i}\partial_t \varrho_{\mathbf{p}} = [\mathsf{H}_{\mathbf{p}}, \varrho_{\mathbf{p}}]$, where the Hamiltonian is [17]

$$\mathsf{H}_{\mathbf{p}} = \Omega_{\mathbf{p}} + \mathsf{V} + \sqrt{2}\, G_{\mathrm{F}} \int \frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3}\, (\varrho_{\mathbf{q}} - \bar{\varrho}_{\mathbf{q}})\, (1 - \mathbf{v}_{\mathbf{q}} \cdot \mathbf{v}_{\mathbf{p}}), \tag{1}$$

$\mathbf{v}_{\mathbf{p}}$ being the velocity. In the mass basis, the matrix of vacuum oscillation frequencies is $\Omega_{\mathbf{p}} = \mathrm{diag}(m_1^2, m_2^2, m_3^2)/2E$. The matter effect is represented, in the weak interaction basis, by $\mathsf{V} = \sqrt{2}\, G_{\mathrm{F}} n_B\, \mathrm{diag}(Y_e, 0, Y_\tau^{\mathrm{eff}})$. For antineutrinos the only difference is $\Omega_{\mathbf{p}} \to -\Omega_{\mathbf{p}}$. The effective tau-lepton density $Y_\tau^{\mathrm{eff}} \approx 10^{-5}$ arises from radiative corrections [18] and can be important in a genuine three-flavor treatment of ordinary [19] or collective [15] SN neutrino oscillations. For the latter, the influence of $Y_\tau^{\mathrm{eff}}$ can be rather sensitive to deviations from maximal 23-mixing.

The angular factor $(1 - \mathbf{v}_{\mathbf{q}} \cdot \mathbf{v}_{\mathbf{p}})$ in Eq. (1) derives from the current-current nature of the weak-interaction Hamiltonian. In an isotropic ensemble it averages to unity, whereas the neutrinos streaming off a SN core are strongly non-isotropic so that different angular modes experience a different strength of the neutrino-neutrino interaction potential. One may expect that this effect leads to kinematical decoherence among angular modes and thus to flavor equilibrium [1, 2]. In a symmetric ensemble with equal densities of neutrinos and antineutrinos, this is indeed the case. Such a system is highly unstable in that an infinitesimal deviation from exact isotropy is enough to trigger an exponential run-away towards flavor equilibrium [7]. On the other hand, a numerical simulation of the flavor evolution of SN neutrinos revealed that multi-angle effects were small: All angular modes evolved nearly collectively, very similar to an isotropic ensemble [4]. Likewise, in a multi-energy system every energy mode feels a different Hamiltonian, yet the evolution is collective, i.e., multi-energy effects do not lead to kinematical decoherence.

We have performed a numerical exploration of multi-angle effects in a spherically symmetric system where the neutrinos are emitted from a "neutrino sphere" [8]. We have identified the asymmetry between the $\nu_e$ and $\bar{\nu}_e$ flux as the crucial parameter. Realistic deleptonization fluxes in SNe seem sufficient to suppress multi-angle decoherence. Therefore, in practice multi-angle effects seem to be a subdominant feature of collective SN transformations.

A better understanding of collective oscillations can be developed in the two-flavor case in terms of the usual flavor polarization vectors $\mathbf{P}_{\mathbf{p}}$ to express the matrices $\rho_{\mathbf{p}}$. The equations of motion are $\partial_t \mathbf{P}_{\mathbf{p}} = \mathbf{H}_{\mathbf{p}} \times \mathbf{P}_{\mathbf{p}}$. Ignoring the ordinary matter effect, the "Hamiltonian" is

$$\mathbf{H}_{\mathbf{p}} = \omega \mathbf{B} + \sqrt{2}\, G_{\mathrm{F}} \int \frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3}\, (\mathbf{P}_{\mathbf{q}} - \bar{\mathbf{P}}_{\mathbf{q}})\, (1 - \mathbf{v}_{\mathbf{q}} \cdot \mathbf{v}_{\mathbf{p}}), \tag{2}$$

where $\mathbf{B}$ is a unit vector in flavor space in the "mass direction" and $\omega = \Delta m^2/2E$ the vacuum oscillation frequency. In an isotropic situation where the velocity terms average to zero, the second term is of the form $\mu\, \mathbf{D}$ where the vector $\mathbf{D}$ is the difference between the total neutrino and antineutrino polarization vectors and $\mu = \sqrt{2} G_{\mathrm{F}}\, n_\nu$ the neutrino-neutrino interaction strength. The individual $\mathbf{H}_{\mathbf{p}} = \omega \mathbf{B} + \mu \mathbf{D}$ all lie in a single plane. In the adiabatic limit the polarization vectors follow the Hamiltonians so that they also lie in this co-rotating plane. This observation explains the collective nature of the multi-energy evolution (all polarization vectors stay in the same plane that rotates around $\mathbf{B}$ with a certain frequency $\omega_{\mathrm{c}}$) and explains the spectral splits in that the final $\mathbf{H}_{\mathbf{p}}$ in the co-rotating plane are either aligned or anti-aligned with $\mathbf{B}$ [9, 10]. This picture also illustrates a crucial difference to the symmetric system where initially $\mathbf{D} = 0$ and the overall evolution is purely pendular [5]—there is no co-rotating plane.

In a spherically symmetric situation every mode is characterized by its vacuum oscillation frequency $\omega$ and its angle $\theta$ relative to the radial direction, providing $v = \cos\theta$ as the radial

velocity. Now the individual Hamiltonians are $\mathbf{H_p} = \omega\mathbf{B}+\mu(\mathbf{D}-v_\mathbf{p}\mathbf{F})$ where $\mathbf{F}$ is the flux term of the neutrino ensemble, i.e., the same as $\mathbf{D}$, but every mode weighted with $v_\mathbf{p}$. If all polarization vectors lie in a single plane, then also all $\mathbf{H_p}$ are in that plane so that an evolution in a co-rotating plane is self-consistently possible. Our numerical simulations show that those cases with little kinematical decoherence correspond to the polarization vectors essentially staying in a co-rotating plane with some zenith-angle spread. On the other hand, the vectors $\mathbf{B}$, $\mathbf{D}$ and $\mathbf{F}$ do not have to stay in a single plane and in fact, this appears to be an unstable arrangement. Kinematical multi-angle decoherence corresponds to strong deviations from this coplanar situation. In a spherically symmetric situation, the neutrino-neutrino interaction strength decreases with $r^{-4}$. In a toy model with an artificially slow decrease of $\mu(r)$ one can make the evolution arbitrarily slow and adiabatic. In this case multi-angle decoherence appears to be unavoidable.

It appears that in a realistic SN the collective evolution is slow enough to be essentially adiabatic with regard to the development of a spectral split, but fast enough that the co-planar arrangement of the polarization vectors survives. This picture may provide the key to an analytic understanding of the conditions for multi-angle kinematical decoherence.

The assumption of spherical symmetry of the overall system severely restricts possible solutions. The evolution is a one-dimensional problem along the radial direction. It remains to be studied if this symmetry has an important impact on kinematical decoherence, i.e., if a system with fewer symmetries would decohere more easily.

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